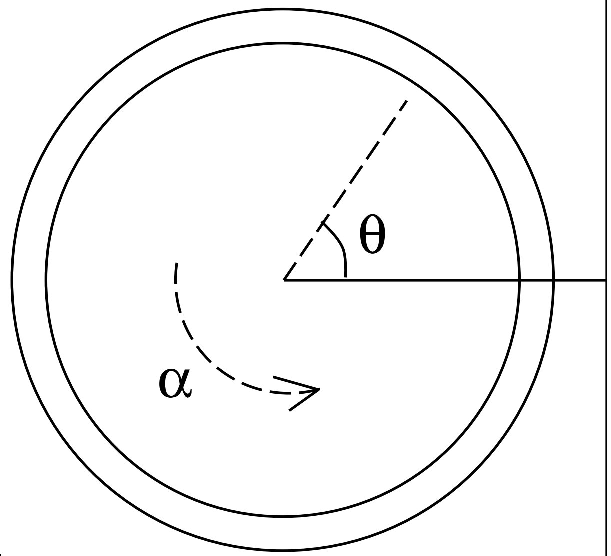
Problem 10.21

A roller is driven by the position function

$$\theta = 2.50t^2 - 0.600t^3$$



a.) What is it's maximum angular velocity?

The angular velocity function is:

$$\omega = \frac{d\theta}{dt} = \frac{d\left(2.50t^2 - 0.600t^3\right)}{dt}$$

$$= 5t - 1.8t^2$$

This will be a maximum when it's derivative equals zero, or when:

$$\frac{d\omega}{dt} = \frac{d\left(5t - 1.8t^2\right)}{dt} = 0$$

$$\Rightarrow \; 5 - 3.6t = 0$$

$$\Rightarrow \; t = 1.39 \text{ s}$$

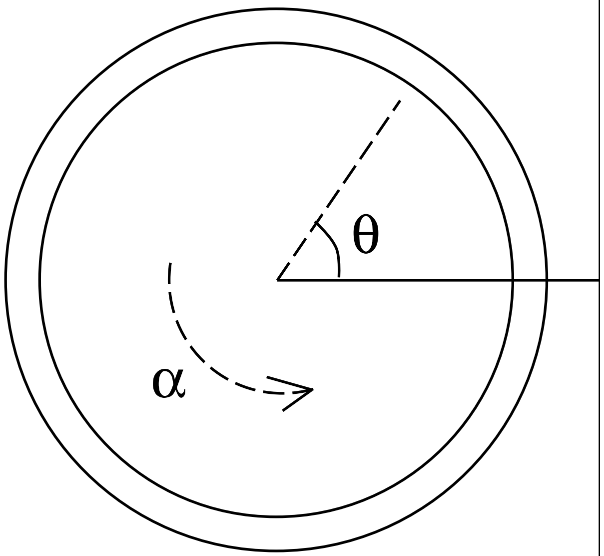
The angular velocity at that time will be:

$$\omega = 5t - 1.8t^2$$

$$= 5(1.39 \text{ s}) - 1.8(1.39 \text{ s})^2 = 3.47 \text{ rad/sec}$$

b.) The maximum tangential speed will happen at the maximum angular velocity, so:

$$\begin{aligned} v_{max} &= R\omega_{max} \\ &= (0.500 \text{ m/rad})(3.47 \text{ rad/sec}) \\ &= 1.74 \text{ m} \end{aligned}$$



c.) The roller will reverse direction when its angular velocity is zero (that's the transit point between "positive" and "negative" angular displacement), so:

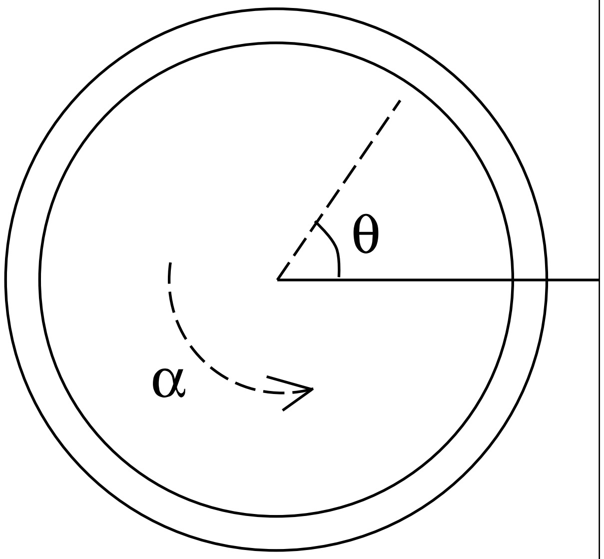
$$\omega = 5t - 1.8t^2$$

is zero at: $5t - 1.8t^2 = 0$

$$\Rightarrow \quad t = 2.78 \text{ s}$$

The motor has to be turned off at the 2.78 second point.

d.) Through how many rotations has the roller turned between its start-up point and the time it turns off?



The time to turn-off is 2.78 seconds. The number of radians rotated through during that period is:

$$\theta = 2.50t^2 - 0.600t^3$$
$$= 2.50(2.78)^2 - 0.600(2.78)^3$$
$$= 6.43 \text{ rad}$$

The number of revolutions this represents is:

$$\#\text{rev} = \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)(6.43 \text{ rad})$$
$$= 1.02 \text{ rotations}$$